

11/1/2020

DATE: / /

PAGE NO: /

Ch-3

* Ex-3.1

Q2 Find a quadratic polynomial, the sum and product of whose zeroes are the following number.

(i) -3, 2

Ans $x^2 - (a + b)x + ab = 0$

$$x^2 - (-3)x + 2 = 0$$

$$x^2 + 3x + 2 = 0$$

(ii) $\sqrt{2}, \frac{1}{3}$

Ans $x^2 - (a + b)x + ab = 0$

$$= x^2 - \sqrt{2}x + \frac{1}{3} = 0$$

Multiply by 3

$$3x^2 - 3\sqrt{2}x + 1 = 0$$

(iii) $-\frac{1}{4}, \frac{1}{4}$

Ans $a + b = \frac{-b}{a} = \frac{-1}{4}$

$$= ab = \frac{c}{a} = \frac{1}{4}$$

$$= a = 4, b = -1, c = 1$$

$$= ax^2 + bx + c = 0$$

$$= 4x^2 - x + 1 = 0$$

(iv) $0, \sqrt{5}$.

$$\text{Ans} = d + \beta = -\frac{b}{a} = 0$$

$$= d + \beta = \frac{c}{a} = \sqrt{5}$$

$$= a = 1, b = 0, c = \sqrt{5}$$

$$= ax^2 + bx + c = 0$$

$$= x^2 + \sqrt{5} = 0$$

(v) $4, 1$

$$\text{Ans} = x^2 - (a + \beta)x + a\beta = 0$$

$$= x^2 - 4x + 1 = 0$$

(vi) $1, 1$

$$\text{Ans} = x^2 - (a + \beta)x + a\beta = 0$$

$$x^2 - 1x + 1 = 0$$

3. If the sum of squares of zeroes of quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, then find the value of k .

Ans. Given polynomial $f(x) = x^2 - 8x + k$
Let α and β are zeroes of polynomial $f(x)$
then.

$$\text{sum of zeroes } (\alpha + \beta) = -\frac{b}{a}$$

$$= -\frac{-8}{1} = 8 \dots \dots \textcircled{1}$$

$$\text{And product of zeroes } (\alpha\beta) = \frac{c}{a} = \frac{k}{1} = k \dots \dots \textcircled{2}$$

Now, from eq. ①

$(\alpha + \beta) = 8$, squaring both sides

$$= (a + b)^2 = 64$$

$$= a^2 + b^2 + 2ab = 64 \quad \text{--- (3)}$$

It is given that sum of squares of 20 is 40. That is, $a^2 + b^2 = 40$

Putting values from eq. (1), (2) in (3)

$$= 40 + 2k = 64$$

$$= 2k = 64 - 40$$

$$= 2k = 24$$

$$= k = 12$$

∴ Thus $k = 12$ Ans

2020

PAGE NO. 4

* Exercise = 3.2.

Q1. Using division algorithm, divide $f(x)$ by $g(x)$ and find quotient and remainder.

(i) $f(x) = 3x^4 + 0x^3 + 0x^2 + 2x + 5$, $g(x) = 1 + 2x + x^2$

Ans

$$\begin{array}{r} x^2+2x+1 \overline{) 3x^4 + 0x^3 + 0x^2 + 2x + 5} \quad \underline{3x-5} \\ \underline{3x^3 + 6x^2 + 3x + 5} \\ -5x^2 - x + 5 \\ \underline{-5x^2 - 10x + 5} \\ 9x + 0 \end{array}$$

(ii) $f(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

Ans

$$\begin{array}{r} x^2-2 \overline{) x^3 - 3x^2 + 5x - 3} \quad \underline{x-3} \\ \underline{x^3 + 2x} \\ -3x^2 + 7x - 3 \\ \underline{-3x^2 + 6} \\ 7x - 9 \end{array}$$

Q: $x-3$, R: $7x-9$

(iii) $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x+2$

Ans

$$\begin{array}{r} x+2 \overline{) x^3 - 6x^2 + 11x - 6} \quad \underline{x^2-8x+27} \\ \underline{-x^3 + 2x^2} \\ -8x^2 + 11x - 6 \\ \underline{-8x^2 + 16x} \\ 27x - 6 \end{array}$$

Q: $x^2 - 8x + 27$,

R = -60.

$27x - 6$

$\underline{-27x + 54}$

$\underline{-60}$

(iv) $9x^4 - 4x^2 + 4$, $g(x) = 3x^2 + x - 1$

Ans

$$\begin{array}{r}
3x^2 + x - 1 \overline{) 9x^4 - 4x^2 + 4} \quad \left[3x^2 - x \right. \\
\underline{-9x^4 + 3x^3 + 3x^2} \\
-3x^3 - x^2 + 4 \\
\underline{-3x^3 - x^2 + x} \\
 + 3x \\
= x + 4
\end{array}$$

Q: $3x^2 - x$, R = $-x + 4$.

Q2 Divide second polynomial by first and verify that first polynomial is a factor of second polynomial.

(i) $g(x) = x^2 + 3x + 2$, $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Ans

$$\begin{array}{r}
 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \quad \left[3x^2 - 4x + 2 \right. \\
\underline{3x^4 + 9x^3 + 3x^2} \\
-4x^3 - 10x^2 + 2x + 2 \\
\underline{-4x^3 - 12x^2 + 4x} \\
 + 2x^2 + 6x + 2 \\
\underline{2x^2 + 6x + 2} \\
 0
\end{array}$$

Q: $3x^2 - 4x + 2$,

R = 0.

(ii) $g(x) = t^2 - 3$, $f(x) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$.

Ans

$$\begin{array}{r}
t^2 - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \quad \left[2t^2 + 3t + 4 \right. \\
\underline{2t^4} \\
 - 6t^2 \\
 - 6t^2 - 9t - 12 \\
 + 6t^2 + 9t + 12 \\
 0
\end{array}$$

$$3t^3 + 4t^2 - 9t - 12$$

$$\underline{3t^3 - 9t}$$

$$4t^2 - 12$$

$$\underline{4t^2 - 12}$$

$$0$$

yes.

$$Q: 2t^2 + 3t + 4,$$

$$R: 0.$$

(iii) $g(x) = x^3 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

Ans

$$x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \quad x^2 - 1$$

$$\underline{x^5 - 3x^3 + x^2}$$

$$-x^3 + 3x + 1$$

$$\underline{-x^3 + 3x - 1}$$

$$2$$

no.

$$Q: x^2 - 1,$$

$$R: 2$$

Q3 In the following polynomial, their zeros are given, find all other zeros

(i) $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2; \sqrt{2}$ and $-\sqrt{2}$

Ans

$$x = \sqrt{2}, x = -\sqrt{2}$$

$$(x - \sqrt{2})(x + \sqrt{2})$$

$$(a - b)(a + b) = a^2 - b^2$$

$$= (x)^2 - (\sqrt{2})^2$$

$$= x^2 - 2$$

$$= \underbrace{x^2 - 2}_{2x^4 - 3x^3 - 3x^2 + 6x - 2} \overline{) 2x^4 - 3x^3 - 3x^2 + 6x - 2}$$

$$\underline{-2x^4 + 4x^2}$$

$$-3x^3 + x^2 + 6x - 2$$

$$\underline{-3x^3 + 6x}$$

$$\begin{aligned}
 & \frac{x^2 - 1}{x^2 - 1} \\
 & \frac{0}{0} \\
 & = 2x^2 - 3x + 1 \\
 & = 2x^2 - 2x - 1x + 1 \\
 & = 2x(x-1) - 1(x+1) \\
 & = 2x(x-1) - 1(x-1) \\
 & = (x-1)(2x-1) \\
 & x = 1, 2x = 1, x = \frac{1}{2}
 \end{aligned}$$

2. $f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35; 2 \pm \sqrt{3}$
 Ans $x = 2 + \sqrt{3}; x = 2 - \sqrt{3}$
 $= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$
 $= (a - b)(a + b) = a^2 - b^2$
 $= (x + 2)^2 - (\sqrt{3})^2$
 $= (x)^2 - 2(x)(2) + (2)^2 - 3$
 $= x^2 - 4x + 4 - 3$
 $= x^2 - 4x + 1$

$$\begin{array}{r}
 \underline{x^2 - 4x + 1} \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \quad \underline{x^2 - 2x - 35} \\
 \underline{-x^4 + 4x^3 - x^2} \\
 -2x^3 - 27x^2 + 138x - 35 \\
 \underline{+ 2x^3 + 8x^2 + - 2x} \\
 -35x^2 + 140x - 35 \\
 \underline{+ 35x^2 + 140x - 35} \\
 0
 \end{array}$$

$$\begin{aligned}
 & = x^2 - 2x - 35 \\
 & = x^2 - 7x + 5x - 35 = 0 \\
 & = x(x-7) + 5(x-7) = 0
 \end{aligned}$$

$$= (x-7)(x+5) = 0$$

$$\therefore x = 7, x = -5$$

$$3 \quad f(x) = x^3 + 13x^2 + 32x + 20; -2$$

$$\text{Ans} = x = -2$$

$$= (x+2) \overline{) \begin{array}{r} x^3 + 13x^2 + 32x + 20 \\ x^3 + 2x^2 \\ \hline 11x^2 + 32x + 20 \\ -11x^2 - 22x \\ \hline 10x + 20 \\ -10x - 20 \\ \hline 0 \end{array} } = x^2 + 11x + 10$$

$$= x^2 + 10x + 1x + 10$$

$$= x(x+10) + 1(x+10)$$

$$= (x+10)(x+1)$$

$$x = -10, x = -1$$

Q4 On dividing polynomial $f(x) = x^3 - 3x^2 + x + 2$ by polynomial $g(x)$ quotient $q(x)$ and remainder $r(x)$ are obtained as $x - 2$ and $-2x + 4$ respectively then find polynomial $g(x)$.

$$\begin{aligned} \text{Ans} &= f(x) = g(x) \cdot q(x) + r(x) \\ &= x^3 - 3x^2 + x + 2 = g(x)(x-2) + (-2x+4) \\ &\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x)(x-2) \\ &\Rightarrow g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x-2} \end{aligned}$$

$$\begin{array}{r} x-2 \overline{) x^3 - 3x^2 + 3x - 2} \\ \underline{x^3 - 2x^2} \\ -x^2 + 3x - 2 \\ \underline{+x^2 - 2x} \\ -x - 2 \\ \underline{+x - 2} \\ 0 \end{array}$$

$$g(x) = x^2 - x + 1$$

* Exercise = 3.3.

Q1 check whether the following equation are quadratic

(i) $x(x+1) + 8 = (x+2)(x-2)$.

Ans $x^2 + x + 8 = x^2 - 2^2$

$= x + 8 = -4$

$= x + 8 + 4 = 0$

$= x + 12 = 0$

$= \frac{1}{2}x + 6 = 0$ No.

(ii) $(x+2)^3 = x^3 - 4$

Ans $= (x)^3 + 3(x)^2(2) + 3x(2)^2 + (2)^3 = x^3 - 4$

$= x^3 + 6x^2 + 12x + 8 = x^3 - 4$

$= 6x^2 + 12x + 8 + 4 = 0$

$= 6x^2 + 12x + 12 = 0$

$= ax^2 + bx + c = 0$ Yes.

(iii) $x^2 + 3x + 1 = (x-2)^2$

Ans $x^2 + 3x + 1 = (x)^2 - 2(x)(2) + (2)^2$

$= x^2 + 3x + 1 = x^2 - 4x + 4$

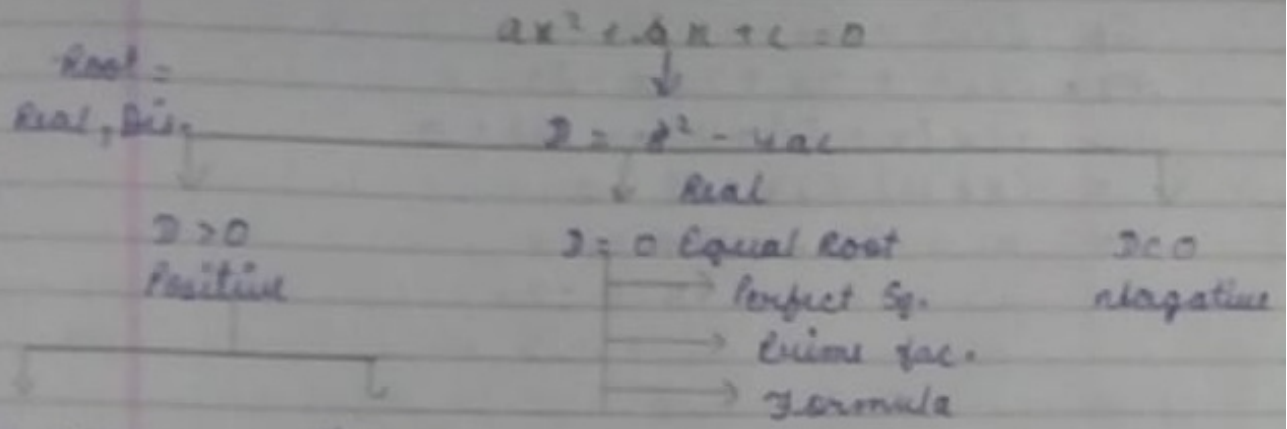
$= 3x + 4x + 1 - 4 = 0$

$= 7x - 3 = 0$

$= 7x - 3 = 0$ No.

(iv) $x + \frac{1}{x} + x^2$

Ans $x^2 + \frac{1}{x} + x^3$ No.



Perfect sq. no. otherwise
 (1, 4, 9, 16, 25, 36) (2, 3, 5, 7)

→ Prime factor → Perfect sq. Method
 → Perfect → Formula method
 → show th. formula

Ex: Prime Factorisation method.

(i) $2x^2 - 5x + 3 = 0$

Ans $= 2x^2 - 2x - 3x + 3 = 0$

$= 2x(x-1) - 3(x-1) = 0$

$= (x-1)(2x-3) = 0$

$= x-1, 2x-3$

$x = \frac{3}{2}$

(ii) $9x^2 - 3x - 2 = 0$

Ans $= 9x^2 - 6x + 3x - 2 = 0$

$3x(3x-2) + 1(3x-2) = 0$

$(3x-2)(3x+1) = 0$

$3x-2 = 0, 3x+1 = 0$

$x = \frac{2}{3}, x = -\frac{1}{3}$

$$\text{(iii)} \quad \sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$$

$$\text{Ans} = \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$$

$$= \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$$

$$= (x + \sqrt{3}) \cdot (\sqrt{3} + 7) = 0$$

$$= x = -\sqrt{3}, \quad \sqrt{3}x = -7$$

$$x = \frac{-7}{\sqrt{3}}$$

$$\text{(iv)} \quad x^2 - 8x + 16 = 0$$

$$\text{Ans} = x^2 - 4x - 4x + 16 = 0$$

$$= x(x - 4) - 4(x - 4) = 0$$

$$= (x - 4)(x - 4) = 0$$

$$= x = 4, \quad x = 4$$

[Q₂] Solve the following equations by factorisation method:

$$\text{(i)} \quad x^2 - 8x + 16 = 0$$

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x} \quad x \neq 2, 1$$

$$\text{Ans} = \frac{x-1 + 2x-4}{(x-2)(x-1)} = \frac{6}{x}$$

$$= \frac{3x-5}{x^2-3x+2} = \frac{6}{x}$$

$$= 3x^2 - 5x + 6 = 6x^2 - 18x + 12$$

$$= 3x^2 - 18x + 12 = 3x^2 + 5x = 0$$

$$= 3x^2 - 13x + 12 = 0$$

$$= 3x^2 - 9x - 4x + 12 = 0$$

$$= 3(x-3) - 4(x-3) = 0$$

$$= (x-3)(3x-4) = 0$$

$$x = 3, \quad 3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

$$(vi) \quad 100x^2 - 20x + 1 = 0$$

$$\text{Ans} = 100x^2 - 10x - 10x + 1 = 0$$

$$= 10x(10x - 1) - 1(10x - 1) = 0$$

$$= (10x - 1)(10x - 1) = 0$$

$$= 10x = 1, \quad 10x = 1$$

$$\therefore x = \frac{1}{10} \quad \text{or} \quad x = \frac{1}{10}$$

$$(vii) \quad 3x^2 - 2\sqrt{6}x + 2 = 0$$

$$\text{Ans} = 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$= \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2})$$

$$= (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2})$$

$$= \sqrt{3}x = \sqrt{2}$$

$$x = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\sqrt{3}$$

$$(viii) \quad x^2 + 8x + 7 = 0$$

$$\text{Ans} = x^2 + 7x + 1x + 7 = 0$$

$$= x(x+7) + 1(x+7) = 0$$

$$= (x+7)(x+1) = 0$$

$$= x = -7, \quad x = -1$$

$$(ix) \quad \frac{x+3}{x+2} = \frac{3x-7}{2x-3}$$

$$\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$$

$$\text{Ans} = (x+3)(2x-3) = (x+2)(3x-7)$$

$$= 2x^2 - 3x + 6x + 9 = 3x^2 - 7x + 6x + 14$$

$$\begin{aligned}
 &= 3x^2 - 7x - 14 - 2x^2 + 3x + 9 = 0 \\
 &= x^2 - 4x - 5 = 0 \\
 &= x^2 - 5x + 1x - 5 = 0 \\
 &= x(x-5) + 1(x-5) = 0 \\
 &= (x-5)(x+1) = 0 \\
 &= x = 5, x = -1
 \end{aligned}$$

$$\begin{aligned}
 \text{(X) } &4x^2 - 4a^2x + a^4 - b^4 = 0 \\
 \text{Ans} &= 4x^2 - 4a^2x + [(a^2)^2 - (b^2)^2] = 0 \\
 &= 4x^2 - 4a^2x + (a^2 + b^2)(a^2 - b^2) = 0 \\
 &= 4x^2 - 2x(a^2 + b^2) - 2x(a^2 - b^2) + (a^2 + b^2)(a^2 - b^2) \\
 &= 2x[2x - (a^2 + b^2) - (a^2 - b^2)] + (a^2 + b^2)(a^2 - b^2) = 0 \\
 &= [2x - (a^2 + b^2)] [2x - (a^2 - b^2)] = 0 \\
 &= 2x = a^2 + b^2 \\
 &= x = \frac{a^2 + b^2}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(xi) } &abx^2 + (a^2 - ac)x - bc = 0 \\
 \text{Ans} &= abx^2 + b^2x - acx - bc = 0 \\
 &= bx(ax + b) - c(ax + b) = 0 \\
 &= (ax + b)(bx - c) = 0 \\
 &= ax = -b, bx = c \\
 &= x = -\frac{b}{a} \quad x = \frac{c}{b}
 \end{aligned}$$

* Exercise → 3.4

1

± Solve the following quadratic equations by the method of perfect square

(ii) $3x^2 - 5x + 2 = 0$

Ans $3x^2 - 5x = -2$

Divide by 3 ÷

$$x^2 - \frac{5}{3}x = -\frac{2}{3}$$

Coeff. of $x = \frac{5}{3}$, half of $\frac{5}{3} \times \frac{1}{2} = \frac{5}{6}$

square = $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$ add both side

$$x^2 - \frac{5}{3}x + \frac{25}{36} = -\frac{2}{3} + \frac{25}{36}$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$(x)^2 - 2x\left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 = \frac{-24 + 25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{1}{36}$$

$$x - \frac{5}{6} = \pm \sqrt{\frac{1}{36}} = \pm \frac{1}{6}$$

$$x = \frac{5}{6} \pm \frac{1}{6}$$

$$x = \frac{5 \pm 1}{6}$$

$$x = \frac{6}{6} \quad x = \frac{4}{6}$$

$$x = 1 \quad x = \frac{2}{3}$$

$$(ii) \quad 5x^2 - 6x - 2 = 0$$

$$\text{Ans} \quad 5x^2 - 6x = 2$$

Divide by 5

$$x^2 - \frac{6}{5}x = \frac{2}{5}$$

$$\text{Coff. Adding Both side } \left(\frac{6}{5} \times \frac{1}{2}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$x^2 - \frac{6}{5}x + \frac{9}{25} = \frac{2}{5} + \frac{9}{25}$$

$$(x)^2 - 2 \times x \times \frac{3}{5} + \left(\frac{3}{5}\right)^2 = \frac{10+9}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$x - \frac{3}{5} = \pm \sqrt{\frac{19}{25}} = \pm \frac{\sqrt{19}}{5}$$

$$x = \frac{3}{5} \pm \frac{\sqrt{19}}{5}$$

$$x = \frac{3 \pm \sqrt{19}}{5}$$

$$x = \frac{3 + \sqrt{19}}{5} \quad | \quad x = \frac{3 - \sqrt{19}}{5}$$

$$(iii) \quad 4x^2 + 3x + 5 = 0$$

$$\text{Ans} \quad 4x^2 + 3x = -5$$

$$x^2 + \frac{3}{4}x = \frac{-5}{4}$$

$$4x^2 + 3x = -5$$

$$x^2 + \frac{3}{4}x = \frac{-5}{4}$$

$$\text{Adding Both side } \left(\frac{3}{4} \times \frac{1}{2}\right)^2 = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

$$x^2 + \frac{3}{4}x + \frac{9}{64} = -\frac{5}{4} + \frac{9}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{-80 + 9}{64}$$

$$\left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$$

$$x + \frac{3}{8} = \sqrt{\frac{-71}{64}}$$

Imaginary.

$$(iv) \quad 4x^2 + 4\sqrt{3}x + 3 = 0$$

$$\text{Ans} \quad 4x^2 + 4\sqrt{3}x = -3$$

$$x^2 + \sqrt{3}x = -\frac{3}{4}$$

$$\text{Add Both side} \left(\sqrt{3} \times \frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$x^2 + \sqrt{3}x + \frac{3}{4} = \frac{-3}{4} + \frac{3}{4} = 0$$

$$\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$$x + \frac{\sqrt{3}}{2} = \pm\sqrt{0} = \pm 0$$

$$x = -\frac{\sqrt{3}}{2} \pm 0$$

$$\frac{x = -\frac{\sqrt{3}}{2} \quad | \quad x = -\frac{\sqrt{3}}{2}}{2 \quad \quad \quad 2}$$

$$(v) \quad 2x^2 + x - 4 = 0$$

$$\text{Ans} \quad 2x^2 + x = 4$$

$$\text{Add Both side} \left(\frac{1}{2} \times \frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$$x^2 + x + \frac{1}{4} = 2 + \frac{1}{4}$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{32+1}{16} = \frac{33}{16}$$

$$x + \frac{1}{4} = \pm \sqrt{\frac{33}{16}} = \pm \frac{\sqrt{33}}{4}$$

$$x = -\frac{1}{4} \pm \frac{\sqrt{33}}{4}$$

$$x = \frac{-1 \pm \sqrt{33}}{4}$$

(vi) $2x^2 + x + 4 = 0 \Rightarrow 2x^2 + x = -4$

Ans $x^2 + \frac{x}{2} = -2$

Add both side $\left(\frac{1}{2} \times \frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$$x^2 + \frac{x}{2} + \frac{1}{16} = -2 + \frac{1}{16}$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{-32+1}{16} = \frac{-31}{16}$$

$$x + \frac{1}{4} = \pm \sqrt{\frac{-31}{16}}$$

Imaginary

(vii) $4x^2 + 4bx - (a^2 - b^2) = 0$

$$4x^2 + 4bx = (a^2 - b^2)$$

$$x^2 + bx = \frac{a^2 - b^2}{4}$$

Adding both sides $\left(b + \frac{1}{2}\right)^2 = \left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$

$$x^2 + bx + \frac{b^2}{4} = \frac{a^2 - b^2}{4} + \frac{b^2}{4}$$

$$\left(\frac{x+b}{2}\right)^2 = \frac{a^2 - b^2 + b^2}{4} = \frac{a^2}{4}$$

$$\frac{x+b}{2} = \pm \sqrt{\frac{a^2}{4}} = \pm \frac{a}{2}$$

$$x = \frac{-b \pm a}{2}$$

$$x = \frac{-b \pm a}{2}$$

$$x = \frac{-b + a}{2} \quad \frac{-b - a}{2}$$

$$\left(x = \frac{-b - a}{2}\right)$$

Q2 Find the roots of the following quadratic equations, if they exist, using the quadratic formula of Shridhar Acharya.

(i) $2x^2 - 2\sqrt{2}x + 1 = 0$

Ans $a = 2, b = 2\sqrt{2}, c = 1$

$$D = b^2 - 4ac$$

$$= (2\sqrt{2})^2 - 4 \times 2 \times 1$$

$$= 8 - 8 = 0$$

Real & equal

$$x = \frac{-b}{2a}, \quad \frac{-b}{2a} = x$$

$$x = \frac{-(2\sqrt{2})}{2 \times 2}$$

$$x = \frac{-2\sqrt{2}}{4}$$

$$x = \frac{\sqrt{2}}{2}$$

(ii) $9x^2 + 7x - 2 = 0$

Ans $a = 9, b = 7, c = -2$

$$D = b^2 - 4ac$$
$$= 7^2 - 4 \times 9 \times (-2)$$
$$= 49 + 72$$

$$D = 121$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-7 \pm \sqrt{121}}{2 \times 9}$$

$$x = \frac{-7 \pm 11}{18}$$

18	
$x = \frac{-7+11}{18}$	$x = \frac{-7-11}{18}$
$x = \frac{4}{18} = \frac{2}{9}$	$x = \frac{-18}{18} = -1$

(iii) $x + \frac{1}{x} = 3, x \neq 0$

Ans $x^2 + 1 = 3x$

$$x^2 - 3x + 1 = 0$$

$$a = 1, b = -3, c = 1$$

$$D = b^2 - 4ac$$
$$= (-3)^2 - 4 \times 1 \times 1$$
$$= 9 - 4 = 5$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$(vi) \frac{1}{x} - \frac{1}{x-2} = 3, \quad x \neq 0, 2$$

$$\text{Ans} \quad \frac{x-2-x}{x(x-2)} = 3$$

$$\frac{-2}{x^2-2x} = 3$$

$$3x^2 - 6x = -2$$

$$3x^2 - 6x + 2 = 0$$

$$a = 3, \quad b = -6, \quad c = 2$$

$$D = b^2 - 4ac$$

$$D = (-6)^2 - 4 \times 3 \times 2$$

$$= 36 - 24 = 12$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{12}}{2 \times 3}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$x = \frac{2(3 \pm \sqrt{3})}{2}$$

$$x = 3 \pm \sqrt{3}$$

57. Divide 16. Find two numbers whose sum is 290

Ansⁿ Let 1st No. = x

2nd No. = x + 2

According to question -

$$(x^2) + (x+2)^2 = 290$$

$$x^2 + x^2 + 2 \times x \times 2 + 2^2 = 290$$

$$2x^2 + 4x + 4 - 290 = 0$$

$$2x^2 + 4x - 286 = 0$$

$$x^2 + 2x - 143 = 0$$

$$a = 1, b = 2, c = -143$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 2^2 - 4 \times 1 \times (-143) \\ &= 4 + 572 = 576 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{-2 \pm \sqrt{576}}{2 \times 1} \end{aligned}$$

$$x = \frac{-2 \pm 24}{2}$$

	$x = \frac{-2+24}{2}$	$x = \frac{-2-24}{2}$
1st	2	2
	$x = 22 = 11$	$x = -26 = -13$
2nd	2	2
	$11+2=13$	

4. The difference of two numbers
 And let large no. = x
 small no. = y

3-6 = 3 times

$$x^2 - y^2 = 45 \quad \text{--- (1)}$$

7

$$y^2 = 4x - 1 \quad \text{--- (2)}$$

$$x^2 - 4x - 45 = 0$$

$$a = 1, b = -4, c = -45$$

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4 \times 1 \times (-45) = 16 + 180 = 196$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{196}}{2 \times 1}$$

$$x = \frac{4 \pm 14}{2}$$

$$x = \frac{4+14}{2}$$

$$x = \frac{4-14}{2}$$

$$x = \frac{18}{2} = 9$$

$$x = \frac{-10}{2} = -5$$

$$y^2 = 4(9) = 36$$

$$y = \sqrt{36} = 6$$

Q

3 Divide 16 - - - - - smaller part

Ans let be large no. = x

Small no = 16 - x

acc. to ques, $(-x)^2 =$

$$(-x)^2 = 2x^2 = (16-x)^2 + 164$$

$$2x^2 = (16)^2 - 2(16)(x) + x^2 + 164$$

$$2x^2 = 256 - 32x + x^2 + 164$$

$$2x^2 - x^2 + 32x - 420 = 0$$

$$x^2 + 32x - 420 = 0$$

$$x^2 + 42x - 10x - 420 = 0$$

$$x(x+42) - 10(x+42) = 0$$

$$(x+42)(x-10) = 0$$

$$x = -42$$

$$x = 10$$

x

=

$$= 16 - 10$$

$$= 6$$

* Exercise - 3.5,

Q 1 Find the name - - - - - equations

(i) $2x^2 - 3x + 5 = 0$

Ans $a = 2, b = -3, c = 5$

$$D = b^2 - 4ac$$

$$= (-3)^2 - 4 \times 2 \times 5$$

$$= 9 - 40$$

$$D = -31$$

Imaginary Root

(ii) $2x^2 - 4x + 3 = 0$

Ans $a = 2, b = -4, c = 3$

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4 \times 2 \times 3$$

$$= 16 - 24$$

$$= -8$$

Imaginary Root

(iii) $2x^2 + x - 1 = 0$

Ans $a = 2, b = 1, c = -1$

$$D = b^2 - 4ac$$

$$= 1^2 - 4 \times 2 \times (-1)$$

$$= 1 + 8$$

$$= 9$$

Real & Dis. Root.

$$(v) x^2 - 4x + 4 = 0$$

$$\text{Ans } a = 1, b = -4, c = 4$$

$$D = b^2 - 4ac$$

$$= (-4)^2 - 4 \times 1 \times 4$$

$$= 16 - 16$$

$$= 0$$

Real & Equal Root

$$(vi) 2x^2 + 5x + 5 = 0$$

$$\text{Ans } a = 2, b = 5, c = 5$$

$$D = b^2 - 4ac$$

$$= 5^2 - 4 \times 2 \times 5$$

$$= 25 - 40$$

$$D = -15$$

Imaginary Root

$$(vii) 3x^2 - 2x + \frac{1}{3} = 0$$

A

$$\text{Ans } a = 3, b = -2, c = \frac{1}{3}$$

$$D = b^2 - 4ac$$

$$= (-2)^2 - 4 \times 3 \times \frac{1}{3}$$

$$= 4 - 4$$

$$= 0$$

Real & Equal Root :

Q2 find the - - - - - real and equal

(i) $kx(x-2) + 6 = 0$

Ans $kx^2 - 2kx + 6 = 0$

$a = k, b = -2k, c = 6$

$D = b^2 - 4ac = 0$
 $= (-2k)^2 - 4 \times k \times 6 = 0$
 $= 4k^2 - 24k = 0$
 $4k(k-6) = 0$

$4k = 0$	$k - 6 = 0$
$k = \frac{0}{4} = 0$	$k = 6$

(ii) $x^2 - 2(k+1)x + k^2 = 0$

Ans $a = 1, b = -2(k+1), c = k^2$

$D = b^2 - 4ac = 0$
 $= [-2(k+1)]^2 - 4 \times 1 \times k^2 = 0$
 $= (2k+2)^2 - 4k^2 = 0$

$(2k)^2 + 2(2k)(2) + (2)^2 - 4k^2 = 0$
 $4k^2 + 8k + 4 - 4k^2 = 0$

$8k = -4$

$k = \frac{-4}{8} = -\frac{1}{2}$

(iii) $2x^2 + kx + 3 = 0$

Ans $a = 2, b = k, c = 3$

$D = b^2 - 4ac = 0$

$k^2 - 4 \times 2 \times 3 = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$$k = \pm \sqrt{24} = \pm \sqrt{6 \times 4}$$

$$k = \pm 2\sqrt{6}$$

$$k = \pm 2\sqrt{6}, k = -2\sqrt{6}$$

(iv) $(k+1)x^2 - 2(k-1)x + 1 = 0$

Ans $a = k, b = -2, c = k$

$$D = b^2 - 4ac = 0$$

$$= (-2)^2 - 4 \times k \times k = 0$$

$$= 4 - 4k^2 = 0$$

$$+ 4k = + 4$$

$$k^2 = \frac{4}{4}$$

$$k = \pm \sqrt{\frac{4}{4}} = \pm \frac{2}{2}$$

$$k = \frac{2}{2}, k = -\frac{2}{2}$$

Q4 Find the - - - - - distinct roots

(i) $kx^2 + 2x + 1 = 0$

Ans. $D = b^2 - 4ac > 0$

$$a = k, b = 2, c = 1$$

$$2^2 - 4 \times k \times 1 > 0$$

$$4 - 4k > 0$$

$$4 > 4k$$

$$\frac{4}{4} > \frac{4k}{4}$$

$$1 > k$$

$$1 > k$$

(ii) $kx^2 + 6x + 1$

Ans $a = k, b = 6, c = 1$

$$\begin{aligned}
 D &= b^2 - 4ac > 0 \\
 b^2 - 4 \times k \times 1 > 0 \\
 36 - 4k > 0 \\
 -4k > -36 \\
 4k < 36 \\
 k < \frac{36}{4} = 9
 \end{aligned}$$

(iii) $x^2 - kx + 9 = 0$
 Ans $a = 1, b = -k, c = 9$
 $D = b^2 - 4ac > 0$
 $= (-k)^2 - 4 \times 1 \times 9 > 0$
 $k^2 - 36 > 0$
 $k^2 > 36$
 $k = \pm \sqrt{36} = \pm 6$
 $k > 6$
 $k < -6$

Q3 Find the roots
 Ans $x^2 + 5kx + 16 = 0$
 $D = b^2 - 4ac < 0$
 $a = 1; b = 5k, c = 16$
 $(5k)^2 - 4 \times 1 \times 16 < 0$
 $25k^2 - 64 < 0$
 $25k^2 < 64$
 $k^2 < \frac{64}{25}$
 $k < \pm \sqrt{\frac{64}{25}}$
 $k < \pm \frac{8}{5}$

$$k < \frac{8}{5}$$

$$k > -\frac{8}{5}$$

Q5 If roots $- - - - - a + c$
are known that $= 2b = a + c$

$$(a = b - c), b = c - a, c = (a - b)$$

$$D = b^2 - 4ac = 0$$

$$(c - a)^2 - 4(b - c)(a - b) = 0$$

$$= c^2 - 2ac + a^2 - 4(ab - b^2 - ac + bc) = 0$$

$$= c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc = 0$$

$$= \underline{c^2 + 2ac + a^2} - \underline{4bc - 4ab} + \underline{4b^2} = 0$$

$$= (c - a)^2 - 4b(c + a) + 4b^2 = 0$$

$$= \underline{(c + a)^2 - 2(c + a) \cdot 2b + (2b)^2} = 0$$

$$(c + a - 2b)^2 = 0$$

$$= \underline{a^2 + 2ab + b^2}$$

$$(a + b)^2$$

$$= c + a - 2b = \sqrt{0} = 0$$

$$= c + a = 2b$$

$$= 2b = a + c$$

* Exercise - 3.6

Q1 Find the LCM & HCF

(i) $2^4 x^2 y z$ and $2^7 x^4 y^2 z^2$

Ans $2^4 x^2 y z = 2^3 \times 3 \times x^2 \times y \times z$

$2^7 x^4 y^2 z^2 = 3^3 \times x^4 \times y^2 \times z^2$

HCF = $3 \times x^2 \times y \times z$

= $3 x^2 y z$

LCM = $2^3 \times 3^3 \times x^4 \times y^2 \times z^2$

= $8 \times 27 \times x^4 y^2 z^2$

(ii) $x^2 - 3x + 2 = (x-2)(x-1)$

$x^4 + x^3 - 6x^2 = x^2(x+3)(x-2)$

LCM →

$x^2 - 3x + 2$

$x^2 - 2x - 1(x-2)$

$x(x-2) - 1(x-2)$

$(x-2)(x-1)$

HCF → HCF = $(x-2)$

LCM = $x^2(x-1)(x-2)(x+3)$

(iii) $2x^2 - 8$ and $x^2 - 5x + 6$

Ans $2x^2 - 8 = 2(x+2)(x-2)$

$x^2 - 5x + 6 =$

HCF = $(x-2)$

LCM = $2(x+2)(x-2)(x-3)$

= $2(x^2 - 4)(x-3)$

$2x^2 - 8$

$2(x^2 - 4)$

$2(x^2 - 2^2)$

$2(x+2)(x-2)$

(iv) $x^2 - 1$; $(x^2 + 1)(x + 1)$ and $x^2 + x - 1$

Ans $x^2 - 1 = (x + 1)(x - 1)$

$$(x^2 + 1)(x + 1) = (x^2 + 1)(x + 1)$$

$$x^2 + x - 1 = (x^2 + x - 1)$$

$$\text{HCF} = 1$$

$$\text{LCM} = (x + 1)(x - 1)(x^2 + 1)(x^2 + x - 1)$$

$$= (x^2 - 1)(x^2 + 1)(x^2 + x - 1)$$

$$= (x^4 - 1)(x^2 + x - 1)$$

(v) $18(64x^4 + x^3 - x^2)$ and $45(2x^6 + 3x^5 + x^4)$

Ans $18(64x^4 + x^3 - x^2) = 2 \times 3^2 \times x^2(x^2 + 2x + 1)(3x - 1)$

$$45(2x^6 + 3x^5 + x^4) = 5 \times 3^2 \times x^4(x + 1)(2x + 1)$$

$$6x^2 + x - 1 \quad (6x^2 + 3x + 1) \times 9$$

$$6x^2 + 3x - 2x - 1 \quad 2x^2 + 2x + 1x + 1$$

$$3x(2x + 1) - 1(2x + 1) \quad 2x(x + 1) + 1(x + 1)$$

$$(2x + 1)(3x - 1) \quad (x + 1)(2x + 1)$$

$$\text{HCF} = 3^2 x^2 (2x + 1)$$

$$= 9x^2(2x + 1)$$

$$\text{LCM} = 2 \times 5 \times 3^2 \times x^4$$

$$(2x + 1)(x + 1)(3x - 1)$$

$$= 90x^4(x + 1)(3x - 1)(2x + 1)$$

Q2. Find the HCF - - - - - expressions.

(i) $a^3 b^4$, $a b^5$, $a^2 b^8$

Ans. $\text{HCF} = a b^4$

$$\text{LCM} = a^3 b^8$$

(ii) $16x^2 y^3$, $48x^4 z$

Ans $16x^2 y^3 = 2^4 \times x^2 \times y^3$

$$48x^4z = 2^4 \times 3 \times x^4 \times z$$

$$\text{HCF} = 2^4 \times x^2 = 16x^2$$

$$\text{LCM} = 2^4 \times 3 \times x^4 \times y^2 \times z$$

$$= 48x^4y^2z$$

(iii) $x^2 - 7x + 12$; $5x^2 - 10x + 21$ and $x^2 + 2x - 15$

Ans) $x^2 - 7x + 12 = (x-4)(x-3)$

$$5x^2 - 10x + 21 = (x-3)(x-7)$$

$$x^2 + 2x - 15 = (x+5)(x-3)$$

$$\text{HCF} = (x-3)$$

$$\text{LCM} = (x-3)(x-4)(x+5)(x-7)$$

(iv) $(x+3)^2(x+2)$ and $(x+3)(x-2)^2$

Ans) $\text{HCF} = (x+3)(x-2)$

$$\text{LCM} = (x+3)^2(x-2)^2$$

(v) $24(6x^4 - x^3 - 2x^2)$ and $20(6x^6 + 3x^5 + x^4)$

Ans) $24(6x^4 - x^3 - 2x^2) = 24 \times 2^2 \times 3 \times x^2(3x-2)(2x+1)$

$$20(6x^6 + 3x^5 + x^4) = 2^2 \times 5 \times x^4(2x+1)(3x+1)$$

$$6x^2 - x - 2$$

$$6x^2 + 5x + 1$$

$$6x^2 - 4x + 3x - 2$$

$$6x^2 + 3x + 2x + 1$$

$$2x(3x-2) + 1(3x-2)$$

$$3x(2x+1) + 1(2x+1)$$

$$(3x-2)(2x+1)$$

$$(2x+1)(3x+1)$$

$$\text{HCF} = 2^2 x^2 (2x+1)$$

$$= 4x^2 (2x+1)$$

$$\text{LCM} = 2^3 \times 3 (3x-2) 5x^4$$

$$(2x+1)(3x+1)$$

$$= 120x^4 (3x-2)(2x+1)(3x+1)$$

Ex 3
Ans

If

$$u(x) = (x-1)^2, v(x) = (x^2-1)$$

$$u(x) = (x-1)^2$$

$$v(x) = (x^2-1) = (x+1)(x-1)$$

$$\text{HCF} = (x-1)$$

$$\text{LCM} = (x+1)(x-1)^2$$

verify

$$\text{LCM} \times \text{HCF} = u(x) \times v(x)$$

$$(x+1)(x-1)^2 \times (x-1) = (x-1)^2(x^2-1)$$

$$(x-1)^2(x^2-1) = (x-1)^2(x^2-1)$$

Q 3

The product $(x-7)(x^2+8x+12)$ (LCM)

$$\text{Ans } (x-7)(x^2+8x+12)$$

$$\text{HCF} = (x+6)$$

$$\text{LCM} = ?$$

HCF \times LCM = Prod. of two exp.

$$(x+6) \times \text{LCM} = (x-7)(x^2+8x+12)$$

$$\text{LCM} = \frac{(x-7)(x+2)(x+6)}{(x+6)}$$

$$\text{LCM} = (x-7)(x+2)$$

$$= x^2 - 5x - 14$$

Q 5

If HCF of two expressions

$$\text{Ans } \text{HCF} \times \text{LCM} = u(x) \times v(x)$$

$$(x-5)(x^3-19x-30) = u(x) \times v(x)$$

$$(x-5) [(x-5)(x^2+5x+6)] = u(x) \times v(x)$$

$$(x-5) [(x-5)(x+2)(x+3)] = u(x) \times v(x)$$

$$\begin{array}{r} x-5 \sqrt{x^3 - 19x - 30} \quad (x^2 + 5x + 6) \\ -x^3 + 5x^2 \\ \hline 5x^2 - 19x - 30 \end{array}$$

$$\begin{array}{r} 5x^2 - 19x - 30 \\ -5x^2 + 25x \\ \hline -19x - 30 \end{array}$$

$$\cancel{6x - 30}$$

$$\cancel{63 - 30}$$

HCF

(Common fac.

$$\underline{\text{I}} \cdot (x-5)(x+2) = x^2 - 3x - 10$$

$$\underline{\text{II}} \cdot (x-5)(x+3) \\ = x^2 - 2x - 15$$